Encoding word order in complex embeddings

Benyou Wang

Huawei Noah's Ark Lab, Shenzhen, China, 23/04/2020

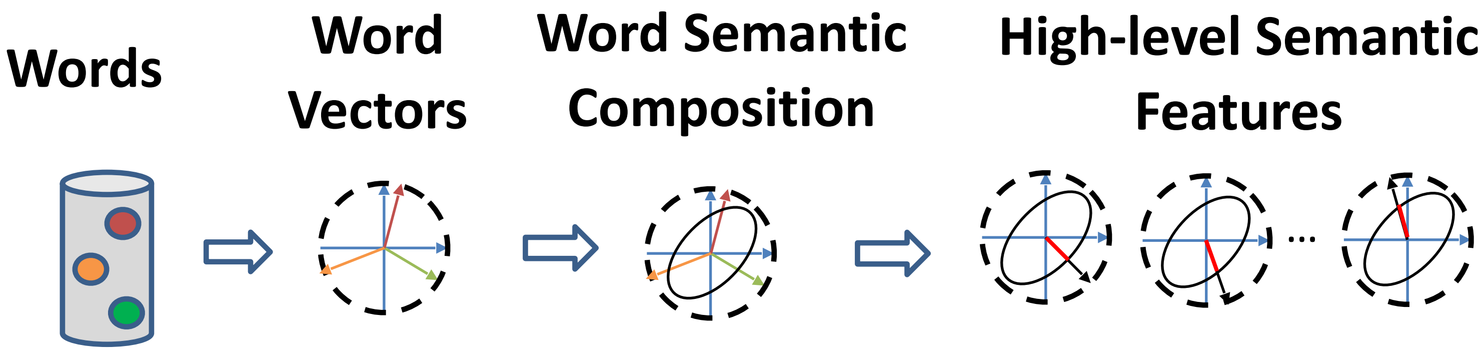
Experience

- Research Experience
- 2014-2017 Master student, Tianjin University, China, supervised by Dawei song.
- 2017-2018 Associate Researcher, Tencent
- 2018-**2021** Marie Curie Researcher and PhD student, University of Padua, supervised by Massimo Melucci

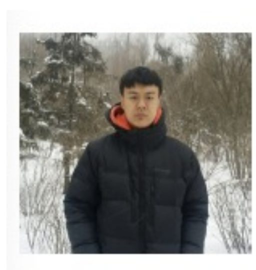
- Visiting
- Sep. - Dec. 2019, University of Copenhagen, hosted by Christina Lioma and Jakob Grue Simonsen
- Dec. - **Feb. 29 2020**, RALI, University of Montreal, hosted by Jian-Yun Nie.

- Awards
- SIGIR 2017 Best paper Honourable mention
- NAACL 2019 Best Explainable NLP Paper

Understanding words: from particles to waves



From set-based probability theory to *vector/projection* based probability theory

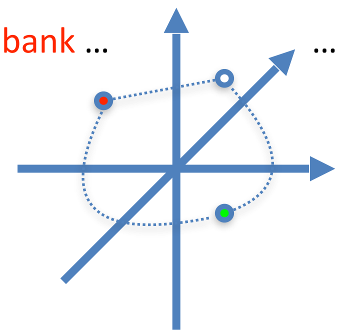


Qiuchi Li*, **Benyou Wang***, Massimo Melucci. A Complex-valued Network for Matching. **NAACL 2019, Best Explainable NLP Paper**

Benyou Wang, Quantum formulations for language: understand words as **particles**, invited talk in Search Engines Amsterdam Meetup, University of Amsterdam, Amsterdam, Netherlands, Oct. 25th. 2019.

Some hints

- Contextualized word embedding [2]

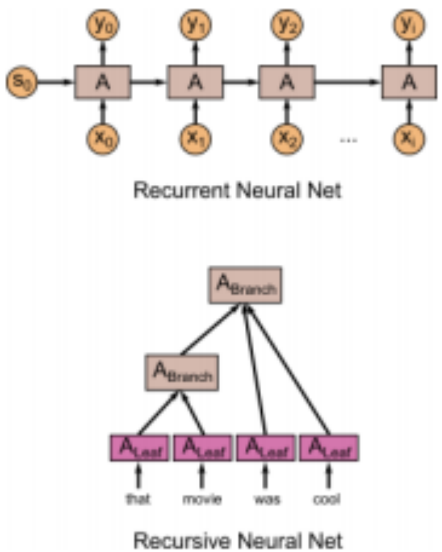


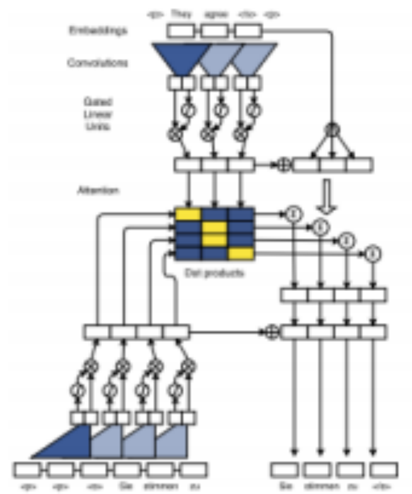
The representation of a word in different contexts might need ***explicitly*** *correlated*

Context: the neighboring words or a simpler case with only **considering word position**

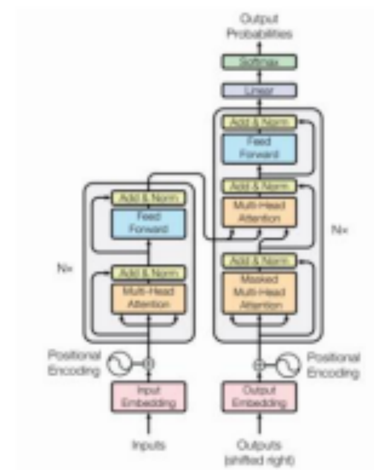
[1] Peters, Matthew E., et al. "Deep contextualized word representations." *NAACL* 2018 best paper.

Why word position is important?





Conv seq2seq



Transformer

Especially If the network structures are insensitive to the word positions (but more efficient in terms of parallelization)!

Position embedding (PE)

Like Word Embedding (WE), PE also admits a map from a position index to a n-dimensional vector $\mathbb{N} \to \mathbb{R}^n$

A word w_j in pos-th position of a sentence is represented as

$$E(w_j, pos) = WE(w_j) + PE(pos) \in \mathbb{R}^n$$

where WE is map from word index to a n-dimensional vector $\mathbb{N} \to \mathbb{R}^n$.

PE

Solution 1 (PE vanilla):
Like word vectors, we just randomly initialize L independent vectors for each position and update them in a data-driven way.

Problems:
In this case, position embeddings are independent without considering their order.

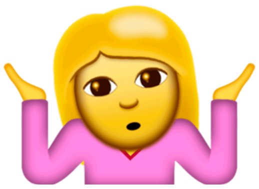
A B C
C B A
A C B
B C A
…

The order of word vocabulary does not matter !

Word index

1 2 3
3 2 1
1 3 2
…

The order of position does matter !



Position index

Gehring, Jonas, et al. "Convolutional sequence to sequence learning." ICML 2017.

TPE (Trigonometric PE)

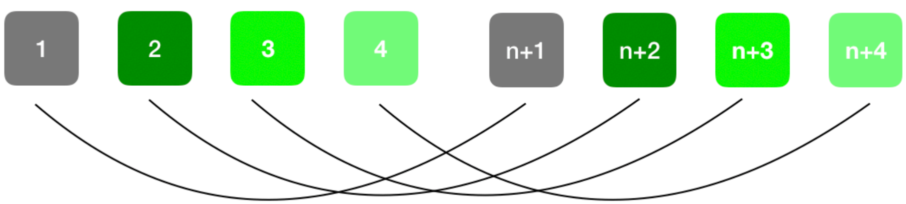
Solution 2 (TPE):

Fixed embedding to capture relative distance:

$$PE'_{2k}(\cdot, pos) = \sin(pos/10000^{2k/d_{model}})$$

$$PE'_{2k+1}(\cdot, pos) = \cos(pos/10000^{2k/d_{model}})$$

Such that the model to easily learn to attend by relative positions, since for any fixed offset k, PE'_{pos+n} can be represented as a linear function of PE'_{pos}, and such function is only parameterized by the offset n . [a.k.a (PFRD) in our paper]



Vaswani, Ashish, et al. "Attention is all you need." *NIPS* 2017.

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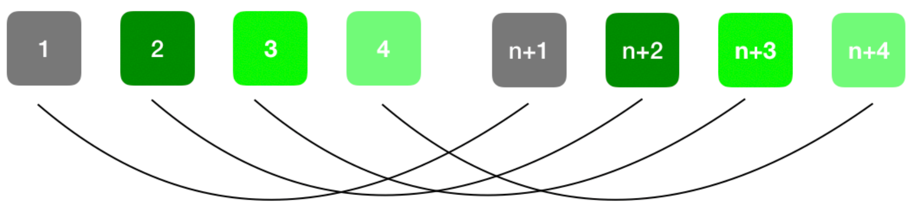

By using $\cos(\alpha+\beta) = \cos(\alpha)cos(\beta) - \sin(\alpha)\sin(\beta)$ and $\cos(\alpha+\beta) = \sin(\alpha)cos(\beta) + \cos(\alpha)\sin(\beta)$

We have $$\begin{bmatrix} PE'_{2k}(pos+n) \\ PE'_{2k+1}(pos+n) \end{bmatrix} = \begin{bmatrix} PE'_{2k+1}(n) & PE'_{2k}(n) \\ PE'_{2k}(n) & -PE'_{2k+1}(n) \end{bmatrix} \times \begin{bmatrix} PE'_{2k}(pos) \\ PE'_{2k+1}(pos) \end{bmatrix}$$

That is $$PE(pos+n) = \begin{bmatrix} PE'_{2k}(pos+n) \\ PE'_{2k+1}(pos+n) \end{bmatrix} = PE(pos) \times f(n)$$

Vaswani, Ashish, et al. "Attention is all you need." *NIPS* 2017.

Problems of TPE

Problems:
1. Can not be trained ! Because it can not have potential to capture relative distance after training.

2. Regarding the ***necessity***
sufficiency : From TPE to PFRD — Done
necessity : From PFRD to TPE —Not yet
Any other solutions or general solutions?

Problem

- The current position embedding **either** can capture relative relationship **or** be trainable!

- We want to propose a position embedding that can be **both** trainable **and** capture relative relationship

Word vectors to word functions

Extending embedding from a **vector** to a **continuous function** over variable the position (pos)

Word index

Technically, $f:(\mathbb{N}, {\color{red}\mathbb{N}}) \rightarrow \mathbb{R}^k$ To $f:(\mathbb{N}) \rightarrow (g: {\color{red}\mathbb{N}} \rightarrow \mathbb{R}^k)$

position index

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position index

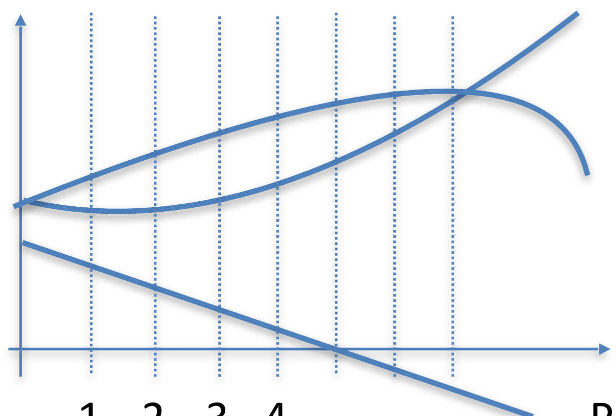
More specifically, we make each dimension independent for simplicity:

$$f:(\mathbb{N}) \rightarrow \{g_i:\mathbb{N} \rightarrow \mathbb{R}\}_{i=1}^{k}$$

Word vectors to word functions

$f:(\mathbb{N}) \to \{g_i:\mathbb{N} \to \mathbb{R}\}_{i=1}^{k}$ One word, as many functions of variable position

For a word

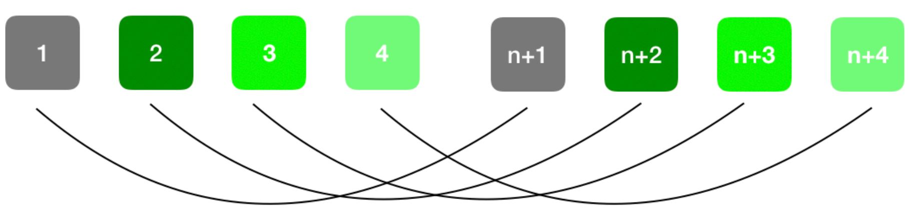


How to decide the function: e.g. boundedness, parameterised by few parameters

Desiderata for word functions

Now, for a specific word w, we have to get it embedding over all the positions, namely a function $\boldsymbol{g_{w,d}}: \boldsymbol{N} \rightarrow \boldsymbol{R}^{\boldsymbol{k}}$

Property 1: Position-free relative-distance (PFRD) transformation
The word/position indexes are invisible in neural networks. It is easier if all the transformation pairs (move a word from one position to another one)
$[g_{w,d}(1) \rightarrow g_{w,d}(n+1), g_{w,d}(2) \rightarrow g_{w,d}(n+2), \cdots, g_{w,d}(L) \rightarrow g_{w,d}(n+L)]$
correspond to a same n-offset-transformation without considering the start position.



Property 2: Boundedness
The function $\boldsymbol{g_{w,d}}$ should be bounded, in order to model long enough sentence

The first property make the problem much simper and can be feasible to solve

Property 1

Problem: We consider the simplest case when the n-offset transformation

$f(n): g(pos) \rightarrow g(n + pos)$

Which transform one from pos-th position to (pos+n) position to be **linear**.

$g_{w,d}(pos) f_{w,d}(n_1) f_{w,d}(n_2) = g_{w,d}(pos) f_{w,d}(n_1 + n_2)$

Solution: It is trivial to get the following solution (proof in the paper):

$f_{w,d}(n) = z_1^{\mathrm{n}}$

Result: z_1 is the parameters and $g_{w,d}(0) = z_2$ [1], such that

$g_{w,d}(pos) = z_2 z_1^{\mathbf{pos}}$;

[1] z_1, z_2 are related to the word index and position index, but superscripts are ignored for simplicity

Property 2

To make $g_{w,d}(pos)$ to be bounded:

$g_{w,d}(pos) = z_2 z_1^{\text{pos}}$; subject to $|z_1| \leq 1$

In real-domain, we necessary consider the extra constraint with some costs.

But if we extend z_1 in complex domain ($x = \alpha + \beta i = re^{i\theta}$), it is easier.

For example, $i = i; i^2 = -1; i^3 = -i; i^4 = 1; \cdots$

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For example, $i = i; i^2 = -1; i^3 = -i; i^4 = 1; \cdots$

Its norm equals 1 always

Let $z_1 = r_1 e^{i\theta_1}; z_2 = r_2 e^{i\theta_2}$

$g_{w,d}(pos) = z_2 z_1^{\text{pos}} = r_2 e^{i\theta_2}(r_1 e^{i\theta_1})^{\text{pos}} = r_2 r_1^{\text{pos}} e^{i(\theta_2+\theta_1 \text{pos})}$ subject to $|r_1| \leq 1$

We directly make $r_1 = 1$, get

$g_w(pos) = r_2 e^{i(\theta_2+\theta_1 \text{pos})}$

The proposed embedding

Our definition:

A word in *pos*-th position is represented as

$$[r_{j,1}e^{i(\omega_{j,1}\text{pos}+\theta_{j,1})}, \ldots, r_{j,2}e^{i(\omega_{j,2}\text{pos}+\theta_{j,2})}, \cdots, r_{j,D}e^{i(\omega_{j,D}\text{pos}+\theta_{j,D})}]$$

where each dimension like d has an amplitude $r_{j,d}$, and a unique period of $p_{j,d} = \dfrac{2\pi}{\omega_{j,d}}$.

i is the imaginary number.

Based on Euler's formula (i.e. $e^{ix} = \cos x + i\sin x$), each element can be rewritten as:

$g_{j,k} = r_{j,d}\cos(\omega_{j,d}\text{pos} + \theta_{j,d}) + r_{j,d}\sin(\omega_{j,d}\text{pos} + \theta_{j,d})i$

Link to TPE

TPE definition: $g'_{j,k} = WE'(j, \cdot\,) + PE'(\,\cdot\,, pos)$

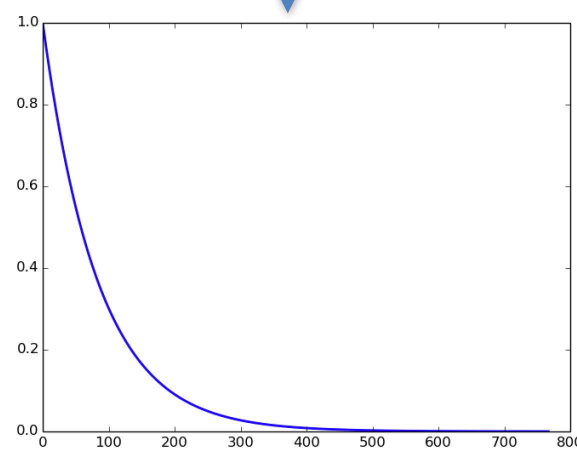
$PE'_{2k}(\,\cdot\,, pos) = \sin(pos/10000^{2k/d_{model}});$

$PE'_{2k+1}(\,\cdot\,, pos) = \cos(pos/10000^{2k/d_{model}})$

It can be considered as a **specific case of ours** when $\omega_{\cdot,d} = \dfrac{1}{10000^{d/2d_{model}})}$

$$g_{j,k} = WE(j) \odot \left(\cos(\omega_{j,d}\text{pos}) + i\sin(\omega_{j,d}\text{pos})\right)$$

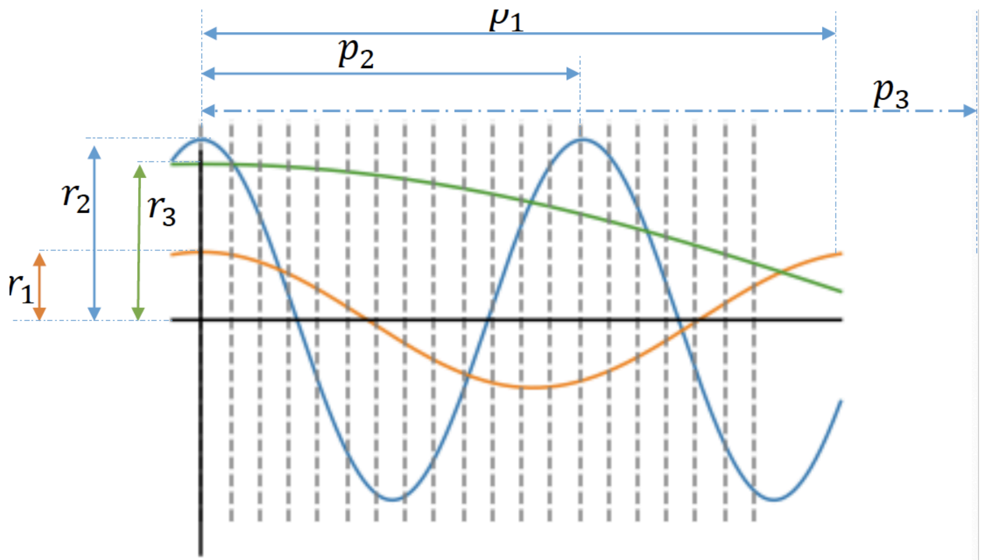
$$g_{j,k} = WE(j) \odot \left(PE'_{2k+1}(\,\cdot\,, pos) + iPE'_{2k}(\,\cdot\,, pos)\right)$$



$\odot$ is the element-wise multiplication

We argue that our proposed embedding is more general.

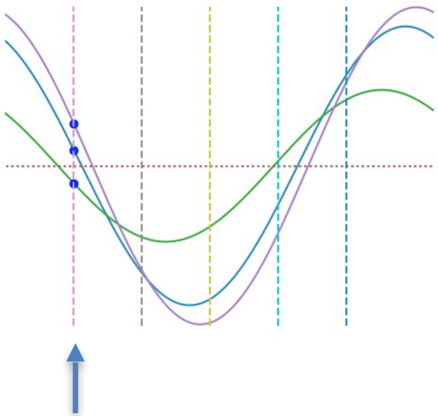
Example of proposed embedding



3-dimensional complex embedding for a single word in different positions. The three wave functions (setting the initial phases as zero) show the real part of the embedding. The x-axis denotes the absolute position of a word and the y-axis denotes the value of each element in its word vector. Colours mark different dimensions of the embedding. The three cross points between the functions and each vertical line (corresponding to a specific position pos) represent the embedding for this word in the pos-th position.

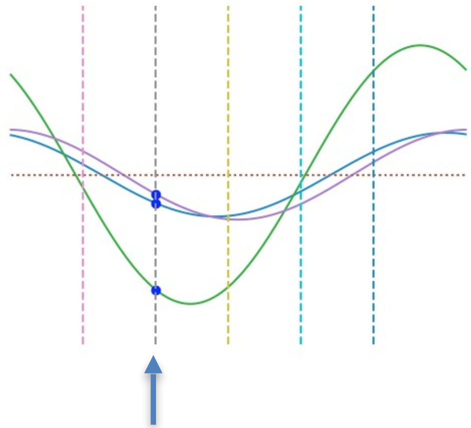
Words as waves

Word functions for ‘I'



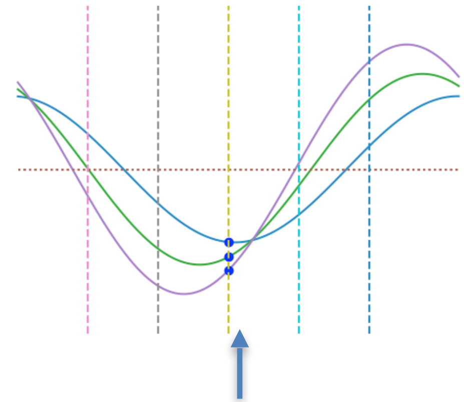
‘I’ is in the 1st position

Word functions for ‘love’



‘love’ is 2nd

Word functions for ‘Montreal’



‘Montreal’ is 3th

For the sentence ‘I love Montreal’

Talking is cheap !

```
import torch
import math
class ComplexNN(torch.nn.Module):
    def __init__(self, opt):
        super(ComplexNN, self).__init__()
        self.word_emb = torch.nn.Embedding(opt.n_token, opt.d_model)
        self.frequency_emb = torch.nn.Embedding(opt.n_token, opt.d_model)
        self.initial_phase_emb = torch.nn.Embedding(opt.n_token, opt.d_model)

    def get_embedding(self, x):

        amplitude = self.word_emb(x)
        frequency = self.frequency_emb(x)
        self.initial_phase_emb.weight = torch.nn.Parameter(self.initial_phase_emb.weight
            % (2 * math.pi))

        sent_len=x.size(-1)
        pos_seq = torch.arange(1, sent_len + 1, 1.0,device=amplitude.device)

        pos_seq = pos_seq.unsqueeze(0).unsqueeze(-1)
        pos_seq = pos_seq.repeat([x.size(0),1,amplitude.size(-1)])

        dimension_bais = self.initial_phase_emb (x)

        enc_output_phase = torch.mul(pos_seq,frequency)+ dimension_bais
        enc_output_real = amplitude * torch.cos(enc_output_phase)
        enc_output_image = amplitude * torch.sin(enc_output_phase)
        # return torch.cat([enc_output_real, enc_output_image],-1)
        return enc_output_real,enc_output_image

    def forward(self, x) :
        return self.get_embedding(x)
```

Applications

- For general neural networks
  - Complex valued neural networks [1,2]
  - Concat real and imaginal -part embedding
- For Transformer
  - Complex Transformer

Trabelsi, Chiheb, et al. "Deep complex networks." *arXiv preprint arXiv:1705.09792* (2017). ICLR 2018

Wolter, Moritz, and Angela Yao. "Complex gated recurrent neural networks." *NIPS* 2018.

Performance -1

In text classification

Method	MR	SUBJ	CR	MPQA	SST	TREC
Fasttext	0.765	0.916	0.789	0.874	0.788	0.874
Fasttext-PE	0.774	0.922	0.789	0.882	0.791	0.874
Fasttext-TPE	0.776	0.921	0.796	0.884	0.792	0.88
Fasttext-Complex-vanilla	0.773	0.918	0.79	0.867	0.803	0.872
Fasttext-Complex-order	**0.787**§†‡*	**0.929**§†‡*	**0.800**§†‡*	**0.889**§†‡*	**0.809**§†‡*	**0.892**§†‡*
LSTM	0.775	0.896	0.813	0.887	0.807	0.858
LSTM-PE	0.778	0.915	0.822	0.889	0.811	0.858
LSTM-TPE	0.776	0.912	0.814	0.888	0.813	0.865
LSTM-Complex-vanilla	0.765	0.907	0.810	0.823	0.784	0.784
LSTM-Complex-order	**0.790**§†‡*	**0.926**§†‡*	**0.828**§†‡*	**0.897**§†‡*	**0.819**§†‡*	**0.869**§†‡*
CNN	0.809	0.928	0.830	0.894	0.856	0.898
CNN-PE	0.816	0.938	0.831	0.897	0.856	0.890
CNN-TPE	0.815	0.938	0.836	0.896	0.838	0.918
CNN-Complex-vanilla	0.811	0.937	0.825	0.878	0.823	0.900
CNN-Complex-order	**0.825**§†‡*	**0.951**§†‡*	**0.852**§†‡*	**0.906**§†‡*	**0.864**§†‡*	**0.939**§†‡*
Transformer w/o position embedding	0.669	0.847	0.735	0.716	0.736	0.802
Transformer-PE	0.737	0.859	0.751	0.722	0.753	0.820
Transformer-TPE (Vaswani et al., 2017)	0.731	0.863	0.762	0.723	0.761	0.834
Transformer-Complex-vanilla	0.715	0.848	0.753	0.786	0.742	0.856
Transformer-Complex-order	**0.746**§†‡*	**0.895**§†‡*	**0.806**§†‡*	**0.863**§†‡*	**0.813**§†‡*	**0.896**§†‡*

Complex vanilla setting refers to the complex-valued word embedding as below:

Wang, Benyou, et al. "Semantic Hilbert Space for Text Representation Learning." *The World Wide Web Conference*. ACM, 2019.

Superscripts §,†,‡and∗mean a significant improvement over a baseline without position embeddings§, PE†, TPE‡and Complex-vanilla∗using Wilcoxon's signed-rank test $p<0.05$

Li, Qiuchi, et al. "Quantum-inspired Complex Word Embedding." *Proceedings of The Third Workshop on Representation Learning for NLP*. 2018.

Performance -2

In machine translation

Table 5.1: Machine translation results. ⋆marks scores reported from other papers.

Method	BLEU
AED (Bahdanau et al., 2014) ⋆	26.8
AED+Linguistic (Sennrich & Haddow, 2016) ⋆	28.4
AED+BPE (Sennrich et al., 2016) ⋆	34.2
Transformer (Ma et al., 2019) ⋆	34.5
Transformer complex vanilla	34.7
Transformer Complex-order	**35.8**

In language model

Table 5.2: Language modeling results. ⋆marks scores reported from other papers.

Method	BPC
BN-LSTM (Cooijmans et al., 2016) ⋆	1.36
LN HM-LSTM (Chung et al., 2016) ⋆	1.29
RHN (Zilly et al., 2017) ⋆	1.27
Large mLSTM (Krause et al., 2016) ⋆	1.27
Transformer XL 6L (Dai et al., 2019)	1.29
Transformer complex vanilla	1.30
Transformer XL Complex-order 6L	**1.26**

Parameter scale

$[r_{j,1}e^{i(\omega_{j,1}\text{pos}+\theta_{j,1})}, \ldots, r_{j,2}e^{i(\omega_{j,2}\text{pos}+\theta_{j,2})}, \cdots, r_{j,D}e^{i(\omega_{j,D}\text{pos}+\theta_{j,D})}]$
For the proposed embedding, there are $3 \times |V| \times D$ parameters in total:
$|V| \times D$ for each $r_{j,d}$, $\omega_{j,d}$, $\theta_{j,d}$.

Initialized phase can be ignored since it is empirically not working.

Two sharing schemas to share parameters:
word-sharing : $\omega_{j,d} = \omega_{\cdot,d}$
dimension-sharing : $\omega_{j,d} = \omega_{j,\cdot}$

Then we can get reasonable parameter scale.

Parameter scale

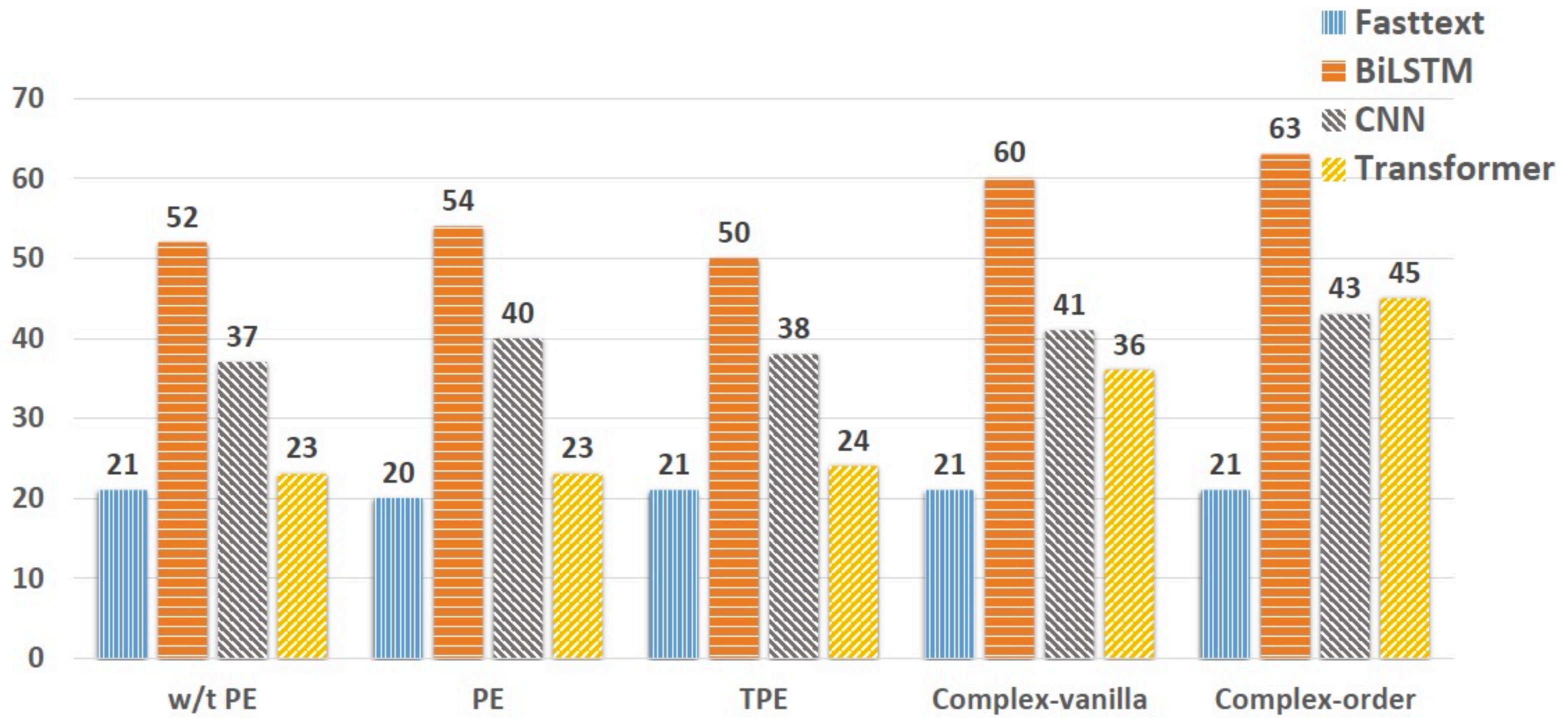
- In transformer

Table 4: Ablation test for Transformer, showing the effect of (i) the definition of embedding layer($f_d(j, \text{pos})$), and (ii) whether the real-part and imaginary transition share the weights, i.e., $\Re(W^{Q/K/V}) = \Im(W^{Q/K/V})$.

Method	Setting		Params	Accuracy	Δ
	$f_d(j, \text{pos})$	share in $W^{Q/K/V}$			
Transformer-complex-order	$r_{j,d}e^{i(\omega_{j,d}\text{pos})}$	×	8.33M	**0.813**	-
adding initial phases	$r_{j,d}e^{i(\omega_{j,d}\text{pos}+\theta_{j,d})}$	×	11.89M	0.785	-0.028
dimension-sharing period schema	$r_{j,d}e^{i\omega_{j,\cdot}\text{pos}}$	×	5.82M	0.797	-0.016
word-sharing period schema	$r_{j,d}e^{i\omega_{\cdot,d}\text{pos}}$	×	5.81M	0.805	-0.008
dimension-sharing amplitude schema	$r_{j,\cdot}e^{i\omega_{j,\cdot}\text{pos}}$	×	5.82M	0.798	-0.015
word-sharing amplitude schema	$r_{\cdot,d}e^{i\omega_{\cdot,d}\text{pos}}$	×	5.81M	0.804	-0.009
w/t encoding positions (complex-vanilla)	$r_{j,d}e^{i\omega_{j,d}}$	×	9.38M	0.764	-0.049
dimension-sharing period schema	$r_{j,d}e^{i\omega_{j,\cdot}\text{pos}}$	✓	4.77M	0.794	-0.019
word-sharing period schema	$r_{j,d}e^{i\omega_{\cdot,d}\text{pos}}$	✓	4.76M	0.797	-0.016
dimension-sharing amplitude schema	$r_{j,\cdot}e^{i\omega_{j,\cdot}\text{pos}}$	✓	4.77M	0.792	-0.021
word-sharing amplitude schema	$r_{\cdot,d}e^{i\omega_{\cdot,d}\text{pos}}$	✓	4.76M	0.801	-0.012
w/t encoding positions (complex-vanilla)	$r_{j,d}e^{i\omega_{j,d}}$	✓	8.33M	0.743	-0.07
vanilla Transformer (Vaswani et al., 2017)	$WE_{j,d} + PE_d$	-	4.1M	0.761	-0.052

Time Cost

Computing time (second per epoch) on TITAN X GPU



Case studies

	words
greatest frequencies in descending order	**worst** solid **stupid powerful** mess **wonderful remarkable** suffers intoxicating **thoughtful** **rare** captures portrait gem frontal **terrific unique** wannabe **witty lousy** **pointless** contrived none **worse refreshingly charming** inventive **amazing junk** incoherent refreshing mediocre **unfunny** thinks **enjoyed heartbreaking delightfully** crisp **brilliant** heart spirit **perfectly** nowhere mistake **engrossing fashioned excellent unexpected wonderfully** means
smallest frequencies in ascending order	slowly proposal schemes roiling juliette titles fabric superstar ah wow choreographed **tastelessness** beg **fabulous** muccino jacobi legendary jae rate example code sensation counter deaths hall eun drug mctiernan storylines cellophane wild motion ups trick comedy entertained mission **frightening** witnesses snoots liners african groan satisfaction calm saturday estranged holm refuses **inquisitive**

Table 7: Words with greatest frequencies and frequencies periods (based on δ_j) in SST (a sentiment classification task), all words are converted to lower-case. The strong sentiment words are bold based on manual labeling.

PE in GCN

setting	MR	SUBJ	CR	MPQA	SST	TREC
GCN	0.786	0.934	0.844	0.833	0.826	0.906
GCN-PE	0.781	0.931	0.810	0.830	0.822	0.884
GCN-TPE	0.548	0.928	0.656	0.828	0.818	0.886
GCN-Complex-vanilla	0.762	0.918	0.831	0.824	0.805	0.886
GCN-Complex-order	0.781	0.931	0.825	0.833	0.816	0.900

GCN also encode structural information (more advanced than positional level) inherently as part of the model, which m

redundant any additional encoding of positional information at the embedding level.

Take-away messages

- Extending word vectors to word functions
- First formal explanation for Trigonometric PE
- A novel approach to embedding can trained to trade off word information and position information
-

Q & A

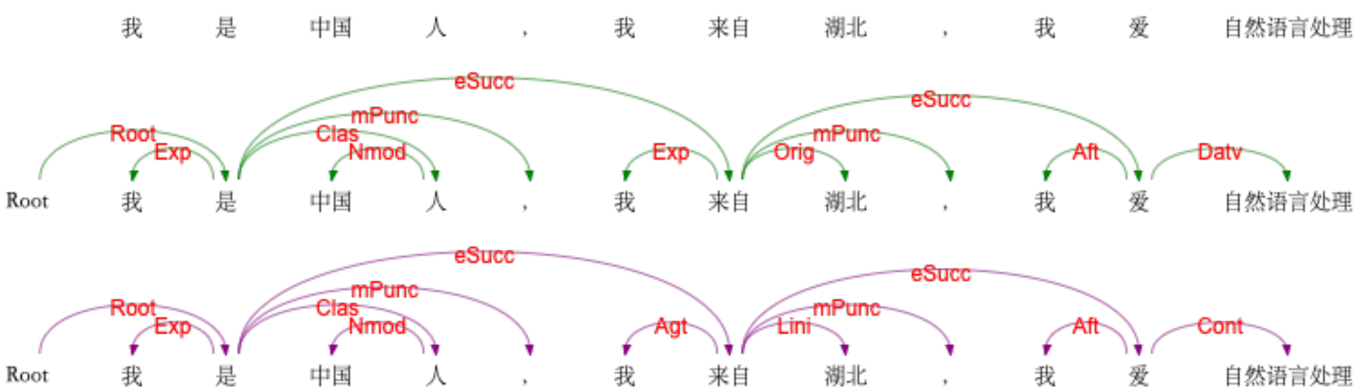
Thanks

wang@dei.unipd.it

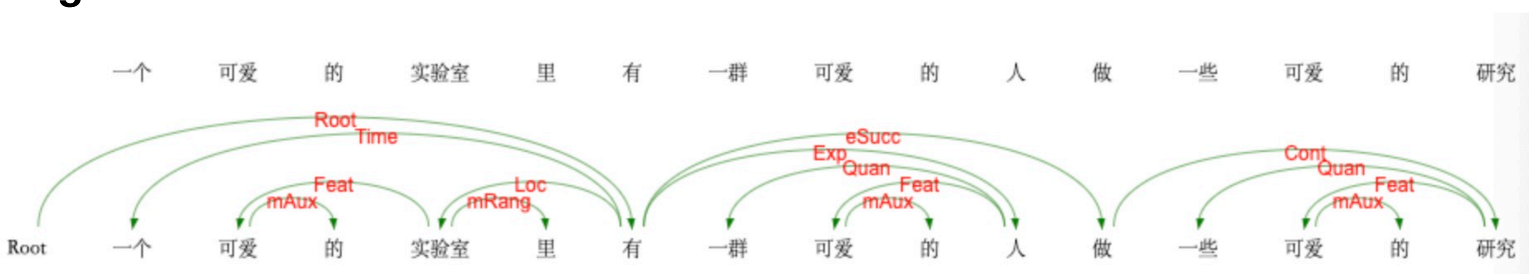
Understanding the periodicity

Periodicity may relate to parsing tree

many sub-sentences



single sentence



Two hinds -1

- Average period is between 14-17 positions, which is close to the length of a sentence.

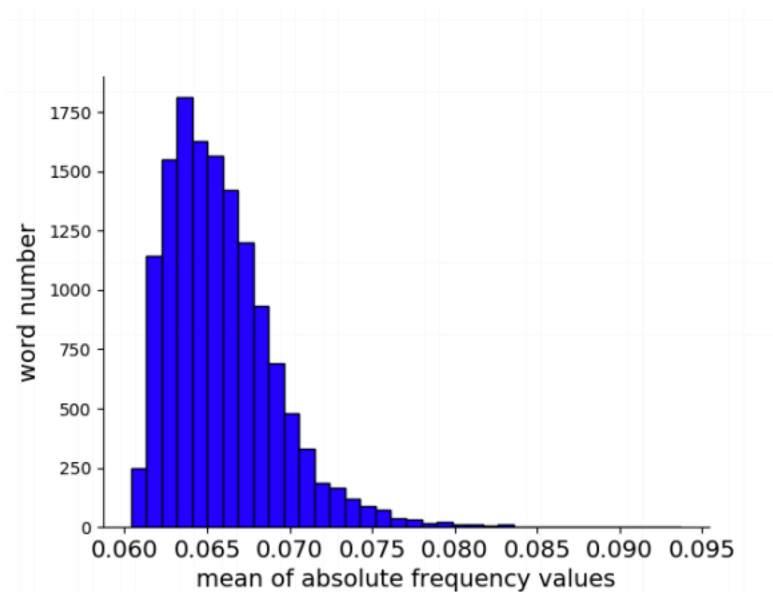


Figure 3: The distribution of the δ_j. Higher values mean that the word representations are more sensitive to the word positions.

Two hinds- 2

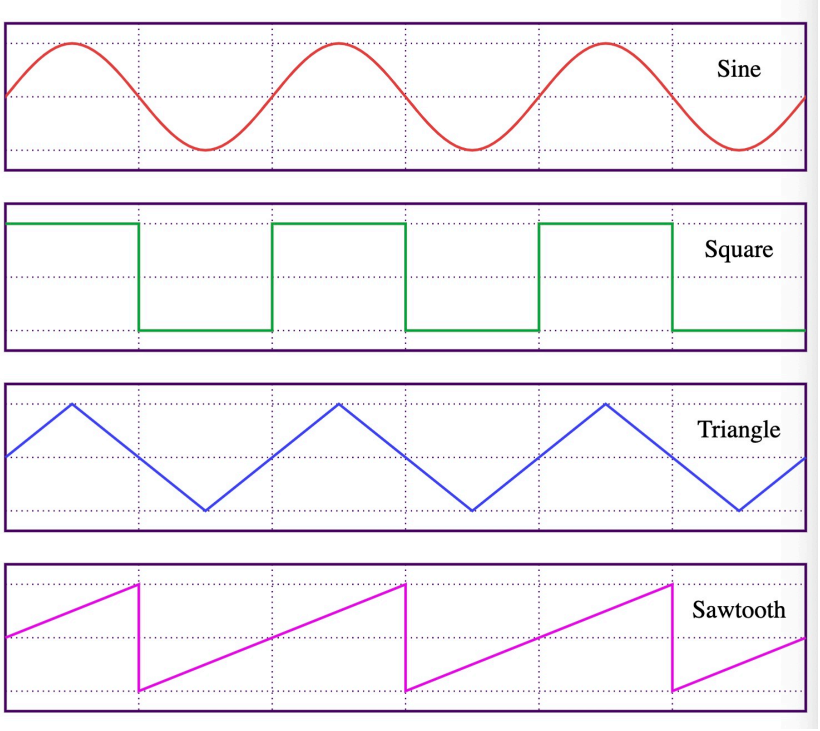
- Define a new qualitative indicator called “ the distance to the local root in the parsing tree”. (DTLR)

- This indicator DTLR will
  – be negative when the word appears **before** its root,
  – become zero when it **is** the root,
  – and will be positive when it appears **after** its rootThis indicator DTLR will

Therefore it behave like a **periodical** indicator across sentences in a document.

Distance to the root.

蒙特利尔大学【root】，是加拿大名列前茅的医博类大学，世界百强大学，国际公立大学论坛成员。
该校建立【root】于1878年，如今已有【root】140年的历史。
近几年来，学校各种学术研究成果在全加拿大综合排名【root】第二。
现有人数52631人，它是【root】世界上最大的法语授课学校。
一个主校区，2个附属学院（工学院和商学院），提供【root】270多个研究生课程。



Adjective words may be more flexible in dependency tree

	words
greatest frequencies in descending order	**worst** solid **stupid powerful** mess **wonderful remarkable** suffers intoxicating **thoughtful** **rare** captures portrait gem frontal **terrific unique** wannabe **witty lousy** **pointless** contrived none **worse refreshingly charming** inventive **amazing junk** incoherent refreshing mediocre **unfunny** thinks **enjoyed heartbreaking delightfully** crisp **brilliant** heart spirit **perfectly** nowhere mistake **engrossing fashioned excellent unexpected wonderfully** means
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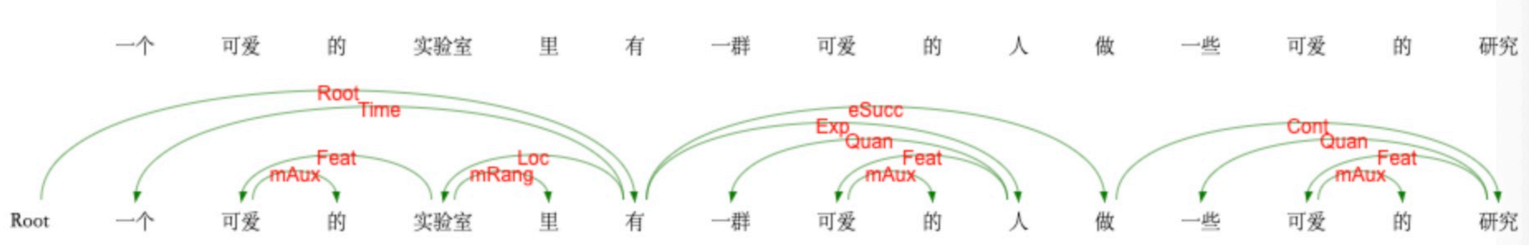

$$PE'_{2k}(pos+n) = PE'_{2k}(pos)PE'_{2k+1}(n) + PE'_{2k+1}(pos)PE'_{2k}(n)$$

$$PE'_{2k+1}(pos+n) = PE'_{2k+1}(pos)PE'_{2k+1}(n) - PE'_{2k}(pos)PE'_{2k}(n)$$

$$\begin{aligned} PE(pos+n) &= [PE'_{2k}(pos); PE'_{2k+1}(pos)] \\ &= PE(pos) \times f(n) \end{aligned}$$